PhD in Economics (XXIth Cycle)
Econometrics test (2019-04-XX)

Name: ____

1. There is some consensus in the medical literature that sleep disorders are more prevalent in night-shift workers. To test it, you decided to run an experiment. You randomly selected 100 workers of a very large firm with thousands of employees, all of them with a night-shift. Then, in agreement with the human resources department, you randomly selected 48 of them and they were exempt from the night-shift ($x_i = 1$). The remaining 52 went on with the usual work-shift ($x_i = 0$). After 6 months you asked all the selected workers if they had had problems in sleeping in the last 3 months. The variable y_i is equal to 1 if worker i reported sleeping problems in the last 3 months and 0 otherwise. The collected data about the outcome variable (y) and the treatment variable (x) are summarized in Table 1.

Table 1: Outcome variable (y) and treatment variable (x)

	$y = 0$	$y = 1$
$x = 0$	24	28
$x = 1$	32	16

Given the probability model based on the following index function model

$$\begin{aligned} y^* &= \alpha + \beta x + \varepsilon \\ y &= \mathbb{1}[y^* > 0], \end{aligned} \tag{1}$$

where $\mathbb{1}$ is the indicator function, equal to 1 if the argument is true:

(a) Prove that the total log-likelihood can be written as

$$\mathcal{L} = 24\log[1 - F(\alpha)] + 28\log[F(\alpha)] + 32\log[1 - F(\delta)] + 16\log[F(\delta)],$$

where $\delta \equiv \alpha + \beta$ and $F(\cdot)$ is the cumulative distribution function of ε.

(b) Estimate by maximum likelihood α and β using the logit model.

(c) Test the hypothesis that $\beta = 0$ by using a likelihood ratio test.

(d) Replicate the hypothesis test on the significance of β after the estimation of the probit model. Does your conclusion about the significance of β change?

(e) Compute the average partial effect of the treatment (exemption from the night-shift) on the probability of having sleep problems. Comment on it.

(f) Calculate the average partial effect of the treatment if a linear probability model is estimated by OLS. Does the average partial effect change?

2. Given a sample of $T = 750$ observations, suppose that the OLS estimates of a VAR(1) model are as follows (standard errors in parentheses).

   Eq. 1: $x_t = \underset{(0.01)}{-0.58} + \kappa\, x_{t-1} - \underset{(0.03)}{0.16} y_{t-1}$,

   Eq. 2: $y_t = \underset{(0.28)}{0.56} + \underset{(0.22)}{0.8}\, x_{t-1} + \underset{(0.12)}{0.36} y_{t-1}$.

   (a) Calculate the unconditional expectation of the process under the assumption $\kappa = 0$.

   $$\begin{bmatrix} E(x_t) \\ E(y_t) \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

   (b) Carry out an appropriate test in order to estabilish if the variable x_t Granger-causes the variable y_t.

   Test type: ____________ Distribution: ________

   Test statistic: ________

   ◯ REJECT ◯ DON'T REJECT

   (c) Re-express the VAR as a VECM using the estimated coefficients.

   $$\begin{bmatrix} \Delta x_t \\ \Delta y_t \end{bmatrix} =$$

   (d) Prove (in a separate sheet) that a process whose coefficients equal the OLS estimates is stationary if $\kappa = 0.5$.

   (e) Find the value $\hat{\kappa}$ for which the estimated VAR(1) is cointegrated.

   $\hat{\kappa} =$ ________________

   (f) Write the cointegration vector when $\kappa = \hat{\kappa}$

   $\beta' =$ [__________ __________]

   (g) Write the loading vector α when $\kappa = \hat{\kappa}$

   $\alpha' =$ [__________ __________]