Phd in Economics - Univpm - Microeconomics - Production theory - 21 July 2016 (3 points per question)

Given the total cost function:

$$\mathbf{c(w_1, w_2, y) = A w_1^{\alpha} w_2^{\beta} y^{\gamma}}$$

where w_1 is the price of the factor x_1, w_2 is the price of the factor x_2, y is output and A, α, β, γ are parameters.

1. define the condition on the parameters such that the cost function is increasing in output;
2. define the condition on the parameters such that the technology is characterized by constant return to scale;
3. define the conditions on the parameters such that the cost function is homogeneous of degree 1 in the price of factors;
4. define the conditions on the parameters such that the cost function is increasing and concave in the price of each factor;
5. compute the ratio between the marginal and the average cost functions;
6. using the Shepard Lemma, compute the conditional demand of the factor x_1, i.e. $x_1(y, w_1, w_2)$
7. compute the elasticity of the conditional labour demand of x_1 with respect to w_1 and to w_2.
8. assume that the firm operates in a competitive market. Which restrictions on the parameters must be applied in order to have a maximum for the profit function?
9. assume $\alpha = \frac{1}{2}, \beta = \frac{1}{2}, \gamma = \frac{3}{2}$ and $A = 1$. Define with p the given price of output. By maximising profits, compute the supply function, i.e. $y(p, w_1, w_2)$.
10. by using the result of point 6 (the conditional labour demand for x_1) and the numerical values of the parameters given in the point 9, compute the unconditional demand of x_1, i.e. $x_1(p, w_1, w_2)$.
11. verify that the unconditional labour demand of factor 1 is homogeneous of degree 0 on the price vector.

Soluzione:

1. $\gamma > 0$
2. $\gamma = 1$
3. $\alpha + \beta = 1$
4. $0 < \alpha < 1, 0 < \beta < 1$
5. $c'_y = \gamma A w_1^\alpha w_2^\beta y^{\gamma-1}$ $\quad \frac{c}{y} = A w_1^\alpha w_2^\beta y^{\gamma-1}$ $\quad \frac{c'_y}{c/y} = \gamma$
6. $x_1 = \alpha A w_1^{\alpha-1} w_2^\beta y^\gamma$
7. $\varepsilon_{x_1,w_1} = \alpha - 1$ $\quad \varepsilon_{x_1,w_2} = \beta$
8. $\gamma > 1$, because marginal cost must be increasing in output.
9. $c(w_1, w_2, y) = (\sqrt{w_1 w_2})\, y^{\frac{3}{2}}$ $\quad \to c'_y = \frac{3}{2}(\sqrt{w_1 w_2})\, y^{\frac{1}{2}}$
   $p = \frac{3}{2}(\sqrt{w_1 w_2})$ $\quad \to y^{\frac{1}{2}}$
   $y = \left(\frac{2}{3}p\right)^2 \frac{1}{w_1 w_2}$
10. $x_1 = \alpha A w_1^{\alpha-1} w_2^\beta y^\gamma$ $\quad \to x_1 = \frac{1}{2}\sqrt{\frac{w_2}{w_1}} y^{\frac{3}{2}}$
    $x_1 = \sqrt{\frac{w_2}{w_1}} \left(\left(\frac{2}{3}p\right)^2 \frac{1}{w_1 w_2}\right)^{\frac{3}{2}}$ $\quad \to x_1 = \frac{8}{27} \frac{p^3}{w_1^2 w_2}$
11. 3-1-2=0